A Ceteris Paribus Borda Solution to the Social Ranking Problem

Extended Abstract

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ABSTRACT

In our society, individuals are often rewarded based on their merits when they work in cooperation. Therefore, we need to design solutions that can fairly rank individuals based on their contribution to the success achieved by alternative groups or coalitions. In this paper, we focus on a novel social ranking solution where individuals are ranked based on the pairwise comparison of coalitions that differ for one single element (referred to in the literature as *Ceteris Paribus* (CP-)comparison). We first introduce a set of axioms inspired by voting theory and social ranking to establish properties that a solution should satisfy when only a limited number of coalition is considered. Then, we show that our set of axioms uniquely characterizes a new solution that mimics a Borda rule computed over a coalitional preorder. These axioms include the one of *desirability*, a very well-established property in the setting of coalitional games but never used before in connection with a Borda rule. The other axioms, specifically *neutrality*, *separability*, and *cancellation*, are properties reflecting eponymous axioms in voting theory.

KEYWORDS

Social Ranking; Coalitional ranking; Borda; Axiomatic; CP-Majority

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1 INTRODUCTION

Starting from groups' comparisons, we face the problem of grounding a ranking of individuals based on the information about the performance of their subsets, and taking into account their merits over multiple subsets. A formalized framework aimed at answering this question is known in the literature as the *social ranking problem* [8, 11, 15]. More precisely, a *solution* for this problem is a method to derive a ranking of individuals based on a transitive relation over a family of coalitions of individuals. Consider for instance the following instance of coalitional preorder on the set of individuals $\{1, ..., 8\}$ (rougly speaking, the symbol '$\succ$' represents 'stronger than' while the symbol '$\sim$' denotes 'equivalent strength'):

$$\{2,3,4\} \succ \{2,6,8\} \succ \{1,3,4\} \succ \{6,3,4\} \sim \{8,3,4\} \succ \{1,6,8\}.$$

Can we reasonably say whether 1 is better than 2 or than 6 by taking into account their multiple positions in the coalitional preorder (where the stronger, the better)?

Solutions to the social ranking problem can be linked to established approaches for addressing similar decision-making problems. Indeed, ordinal Banzhaf solution [13] comes from cooperative game theory [4], whereas CP-majority [11] is inspired by the classic voting rule of Condorcet [9]. We follow this dynamic here, and suggest an adaptation of the well-known Borda's voting rule [21]. Following the approach of the CP-majority solution to the social ranking problem, we adopt the "*Ceteris Paribus*" point of view where a coalition S is regarded as a single voter that compares two individuals i and j through the comparison of $S \cup \{i\}$ and $S \cup \{j\}$.

2 A BORDA SOLUTION

Let N be a finite set. We denote the set of total preorders on N as $\mathcal{R}(N)$ and the set of preorders on subsets of N as $\mathcal{T}(2^N)$.

Definition 2.1. Given any finite subset $N \subseteq \mathbb{N}$, a **solution to the social ranking problem on N** is an application R^N from $\mathcal{T}(2^N)$ to $\mathcal{R}(N)$. A **solution to the social ranking problem** is a set R of solutions to the social ranking on every set of individuals $N = \{1, ..., n\}$, $R = \{R^N : N = \{1, ..., n\}, n \in \mathbb{N}\}$. If R^N is a solution to the social ranking problem on N, for every preorder $\succcurlyeq$ in $\mathcal{T}(2^N)$ we denote by $P^N(\succcurlyeq)$ the asymmetric part of $R^N(\succcurlyeq)$ and $I^N(\succcurlyeq)$ its symmetric part.

In the following, let N be a set of individuals and $\succcurlyeq$ be an element of $\mathcal{T}(2^N)$. We will denote by $\text{supp}(\succcurlyeq)$ the **support** of $\succcurlyeq$:

$$\text{supp}(\succcurlyeq) = \{S \in 2^N : \exists T \in 2^N,\ T \neq S,\ T \succcurlyeq S \text{ or } S \succcurlyeq T\}.$$

The set of **participants** is a set of individuals such that

$$\text{part}(\succcurlyeq) = \{i \in N :\ \exists S \in \text{supp}(\succcurlyeq), i \in S\}.$$

To be able to make a connection with Borda's social preference function [21] we need to define *voters*. The set of **voters** for $\succcurlyeq$ is:

$$\begin{aligned} \text{vot}(\succcurlyeq) \quad = \quad & \{S \in 2^N : \exists i, j \in N, i \neq j \\ & \text{and } S \cap \{i,j\} = \emptyset,\ S \cup \{i\}, S \cup \{j\} \in \text{supp}(\succcurlyeq) \\ & \text{and } S \cup \{i\} \succcurlyeq S \cup \{j\} \text{ or } S \cup \{j\} \succcurlyeq S \cup \{i\}\}. \end{aligned}$$

To each voter S we associate a preorder $\geq^S$ such that, for every i, j in $\text{can}(\succcurlyeq)$, $S \cup \{i\} \succcurlyeq S \cup \{j\} \Leftrightarrow i \geq^S j$. We say that $\geq^S$ is the preference preorder of the voter S.

Similar to the social preference frame [21], we define $\pi_{ij}(\succcurlyeq)$ for every participant i, j as $\pi_{ij}(\succcurlyeq) = |\{S \in \text{vot}(\succcurlyeq), i >^S j\}|$.

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Definition 2.2 (A Borda score). Given a coalitional preorder $\succcurlyeq$ in $\mathcal{T}(2^N)$ and an individual $i \in N$, define i's **Borda score** in $\succcurlyeq$, $\beta^{\succcurlyeq}(i)$:

$$\beta^{\succcurlyeq}(i) = \sum_{j \in N} \pi_{ij}(\succcurlyeq) - \pi_{ji}(\succcurlyeq).$$

Definition 2.3 (CP-Borda). Given a set of individuals $N = \{1, ..., n\}$, the **CP-Borda solution to the social ranking problem on N**, R_B^N is the social ranking solution on N such that for every coalitional preorder $\succcurlyeq$ in $\mathcal{T}(2^N)$ and every individuals i and j in N, $iR_B^N(\succcurlyeq)j$ if $\beta^{\succcurlyeq}(i) \geq \beta^{\succcurlyeq}(j)$. As before, we call $P_B^N(\succcurlyeq)$ the asymmetric part, and $I_B^N(\succcurlyeq)$ the symmetric part of $R_B^N(\succcurlyeq)$. $R_B = \{R_B^N, N = \{1, ..., n\}, n \in \mathbb{N}\}$ is the **CP-Borda solution to the social ranking problem**.

3 AXIOMS AND RESULT

It is common in decision procedures to want to ensure a notion of neutrality towards alternatives or individual's labels. For every element $\succcurlyeq$ of $\mathcal{T}(2^N)$ we denote as $\sigma(\succcurlyeq) \in \mathcal{T}(2^N)$ a coalitional preorder such that $S\sigma(\succcurlyeq)T \Leftrightarrow \sigma(S) \succcurlyeq \sigma(T)$.

Definition 3.1 (Neutrality$_{SR}$). A social ranking solution R satisfies **Neutrality**$_{SR}$ if for every set of individuals N, every element $\succcurlyeq$ of $\mathcal{T}(2^N)$ and every permutation σ on N it holds that

$$\sigma(R^N(\succcurlyeq)) = R^N(\sigma(\succcurlyeq)).$$

Neutrality$_{SR}$ has already been used in axiomatic characterizations of solutions to the social ranking, for instance in [8]. Other formulations exist such as the Neutrality axiom introduced in [11], but CP-Borda does not satisfy this version.

Given two elements of $\mathcal{T}(X)$, $\succcurlyeq$ and $\succcurlyeq'$, we denote by $\succcurlyeq \cap \succcurlyeq'$ the element of $\mathcal{T}(X)$ such that for every i and j in X, $i(\succcurlyeq \cap \succcurlyeq')j$ if $i \succcurlyeq j$ and $i \succcurlyeq' j$. Given a set of individuals N and two elements $\succcurlyeq_1$ and $\succcurlyeq_2$ in $\mathcal{T}(2^N)$, if $\text{supp}(\succcurlyeq_1) \cap \text{supp}(\succcurlyeq_2) = \emptyset$ we can define the set

$$\succcurlyeq_1 \Cup \succcurlyeq_2 = \{\succcurlyeq \in \mathcal{T}(2^{N'}), \text{supp}(\succcurlyeq) = \text{supp}(\succcurlyeq_1) \cup \text{supp}(\succcurlyeq_2), N' \supseteq N \text{ and } \succcurlyeq \supseteq \succcurlyeq_1, \succcurlyeq \supseteq \succcurlyeq_2\}.$$

Definition 3.2 (Separability$_{SR}$). A social ranking solution R satisfies **Separability**$_{SR}$ if for every set of individuals N and every elements $\succcurlyeq_1$, $\succcurlyeq_2$ of $\mathcal{T}(2^N)$ such that there is no coalition S and individuals i, j with $S \cup \{i\}$ is in supp($\succcurlyeq_1$) and $S \cup \{j\}$ is in supp($\succcurlyeq_2$), then for any element $\succcurlyeq$ of $\succcurlyeq_1 \Cup \succcurlyeq_2$, if $\succcurlyeq$ is in $\mathcal{T}(2^{N'})$ it holds that

$$R^{N'}(\succcurlyeq) \supseteq R^N(\succcurlyeq_1) \cap R^N(\succcurlyeq_2).$$

Separability has been introduced (as Consistency) by H.P. Young in [21]. It is now a common considered axiom in the study of voting rules. To combine two coalitional preorders, Separability$_{SR}$ axiom requires that the respective supports of $\succcurlyeq_1$ and $\succcurlyeq_2$ are disjoint, so that we are able to define the union of two preorders, but also that we are neither considering the same voter multiple times nor creating a new voter. Compared to the Consistency axiom introduced and studied by T. Suzuki and M. Horita in [19], these constraints on the structure of the two coalitional preorders $\succcurlyeq_1$ and $\succcurlyeq_2$ reduce the number of situations to which Separability$_{SR}$ applies.

Given a set of individuals N and a coalitional preorder $\succcurlyeq$ in $\mathcal{T}(2^N)$, we say that an individual i of N is **(resp. strictly) more desirable** than an individual j in N if for every voter S in vot($\succcurlyeq$), $S \cap \{i, j\} = \emptyset$, $S \cup \{i\} \succcurlyeq S \cup \{j\}$ (resp. and there is at least one voter T in vot($\succcurlyeq$) such that $T \cup \{i\} \succ T \cup \{j\}$).

Definition 3.3 (Desirability$_{SR}$). A social ranking solution R satisfies **Desirability**$_{SR}$ if for every set of individuals N, every element $\succcurlyeq$ of $\mathcal{T}(2^N)$ and every individuals i, j in N it holds that

$$i \text{ is (resp. strictly) more desirable than } j \Rightarrow iR^N(\succcurlyeq)j \text{ (resp. } iP(\succcurlyeq)j).$$

Desirability is a widely studied notion for coalitional games [12, 14] and the Desirability$_{SR}$ axiom for social ranking solutions is studied in [1] to prioritize an individual that systematically performs better than another individual in all CP-comparisons. This axiom has been used in [1] for the axiomatic characterization of other solutions to the social ranking problem, like CP-majority and other lexicographic solutions.

In the following, for every finite set N, we denote $\sim_N$ the binary relation R such that xRy and yRx for every x, y in N.

Definition 3.4 (Cancellation$_{SR}$). A social ranking solution R satisfies **Cancellation**$_{SR}$ if for every set of individuals N and every element $\succcurlyeq$ of $\mathcal{T}(2^N)$ such that $\pi_{ij}(\succcurlyeq) = \pi_{ji}(\succcurlyeq)$ for every i, j in N, then it holds that

$$R^N(\succcurlyeq) = \sim_N .$$

As far as we know, the property of Cancellation$_{SR}$ has not been previously introduced or used to characterize any other social ranking solution. Its counterpart for voting procedures, Cancellation property, has also been introduced by H. P. Young in [21].

As main result, we prove that CP-Borda is the only solution to the social ranking problem that verifies Neutrality$_{SR}$, Separability$_{SR}$, Desirability$_{SR}$ and Cancellation$_{SR}$. Our proof is divided in three lemmas. For the first two lemmas, we use the proof scheme introduced by B. Hansson and H. Sahlquist in [10], and their heavy use of the notion of amplification of a profile, here transposed to amplification of a coalitional order. These ensure that a solution satisfying our set of axioms only relies on Borda scores of individuals. For the last lemma, we use the idea of S. Nitzan and A. Rubinstein in their article [16] to construct a more usable profile to prove that the final ranking is in the "right order".

4 CONCLUSION

In this paper, we introduce and axiomatically characterize a novel solution for the social ranking problem that mimics the well known Borda's voting rule in a coalitional setting. The definition of this solution is based on the notion of *Ceteris Paribus* comparison for coalitions, that was already presented in related literature [11]. This notion has been used to provide a set of meaningful axioms characterizing our CP-Borda solution. Although the convincing interpretation of the axioms used in this paper suggests that our definition of the CP-Borda solution is sound, we note that alternative ways to define a coalitional Borda score are possible, perhaps even using more general information provided by a coalitional preorder, such as the number of dominated coalitions (which are not necessarily those involved in CP-comparisons).

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